

Increasing Life Expectancy and Pay-As-You-Go Pension Systems

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*The content of these slides reflects the views of the authors and not necessarily those of the OeNB.

Motivation

- Pension systems have to cope with two **two demographic developments**:
 - Increases in **life expectancy**.
 - Fluctuations (and mostly declines) in **fertility**
- I deal with the **first aspect**, since it represents an *ongoing* process with considerable and far-reaching budgetary consequences.

Automatic Adjustment Rules

- “Around half of OECD countries have elements in their mandatory retirement-income provision that provide an **automatic link between pensions and a change in life expectancy** [...] The rapid spread of such life-expectancy adjustments has a strong claim to be the **most important innovation** of pension policy in recent years” (OECD, *Pensions at a Glance*, 2011, p. 82).
- Despite this claim there **does not exist much research** on this “most important innovation”.

NDC systems

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- Fixed **contribution rate**: $\tau(t) = \hat{\tau}$
- Life-time **assessment period**
- Past contributions are revalued with an appropriate **notional interest rate**
- At retirement the **notional capital** is transformed into annual pension payments by taking the development of **life expectancy** into account
- **Advantages**: Close **relation** between contributions and benefits; **flexibility** in retirement age with **automatic reaction** of the pension level to the age of retirement; individual accounts and annual statements increase **transparency** (▶ Orange envelope); transnational portability.

Why focus on NDC?

- It is **increasingly popular** (Sweden and around 10 other countries).
- The World Bank, OECD and European Commission often use it as a reference points or **benchmark** to discuss reforms.
- They are explicitly **designed to deal with increasing life expectancy**.
- **Other systems** (German earnings-point, Austrian “notional defined benefit” system APG) can be **directly related** to it.

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- **Remaining life expectancy** (used to calculate the pension benefit at retirement).

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Conventional wisdom:

- Use the **growth rate of the wage bill** as the notional interest rate:

“Viewed from a macroeconomic perspective, the ‘natural’ rate of return for an NDC system is the implicit return of a PAYG system: that is, the growth rate of the contribution bill” (Börsch-Supan, 2003)
- Use the **cohort (i.e. forecasted) life expectancy**:

“The generic NDC annuity embodies [...] cohort life expectancy at the time the annuity is claimed” (Palmer, 2006).

Main Finding

Main Finding

Both components of the conventional wisdom have to be modified:

- It is sufficient to use **periodic life expectancy** to calculate the pension benefit.
- One should use an **“adjusted growth rate of the wage bill”** as the notional interest rate.

Notation 1

The generation born in period t has:

- cohort size $N(t) = N$
- life expectancy $T(t)$
- retirement age $R(t)$

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NOTE 1: I assume that all members of one generation reach the cohort-specific maximum age $T(t)$.

NOTE 2: The maximum age *observed* in period t is denoted by $\tilde{T}(t)$ and the retirement age by $\tilde{R}(t)$. In general: $T(t) \neq \tilde{T}(t)$ and $R(t) \neq \tilde{R}(t)$.

Notation 2

For generation t the PAYG system stipulates the following income streams:

- Contributions:

$$\tau(t+a)W(t+a) \quad \text{for } 0 \leq a < R(t)$$

- Pensions:

$$P(t, a) \quad \text{for } R(t) \leq a \leq T(t)$$

NOTE: In NDC systems the contribution rate is fixed, i.e.
 $\tau(t) = \hat{\tau}$.

Budget of the PAYG system

For the system in period t :

- Labor force: $L(t) = \tilde{R}(t) \times N$
- Retired population: $B(t) = \left(\tilde{T}(t) - \tilde{R}(t) \right) \times N$
- Average pension: $\bar{P}(t) = \frac{\int_{\tilde{R}(t)}^{\tilde{T}(t)} P(t-a, a) da}{\tilde{T}(t) - \tilde{R}(t)}$
- Dependency ratio: $z(t) = \frac{B(t)}{L(t)} = \frac{\tilde{T}(t) - \tilde{R}(t)}{\tilde{R}(t)}$
- The balanced budget condition is given by:

$$\underbrace{\tau(t)W(t)L(t)}_{\text{Revenue}=I(t)} = \underbrace{\bar{P}(t)B(t)}_{\text{Expenditure}=O(t)}$$

The development of life expectancy

An old controversy—How to best model life expectancy?

- Life expectancy **increases in a linear fashion** [▶ Graph](#):

$$T(t) = T(0) + \gamma \cdot t$$

- Robust relationship: In the data: γ between 0.15 and 0.33.
- From $T(t - \tilde{T}(t)) = \tilde{T}(t)$ it follows that: $\tilde{T}(t) = \frac{1}{1+\gamma} T(t)$.

A formal expression of NDC Systems 1

- The **notional capital** before retirement:

$$K(t, R(t)) = \int_0^{R(t)} \hat{\tau} W(t+a) e^{\int_{t+a}^{t+R(t)} \rho(s) ds} da,$$

where $\rho(s)$ stands for the notional interest rate in period s .

- The **first pension payment**:

$$P(t, R(t)) = \frac{K(t, R(t))}{\Gamma(t, R(t))},$$

where $\Gamma(t, R(t))$ is the remaining life expectancy of generation t at age $R(t)$.

- Existing **pensions are adjusted** according to:

$$P(t, a) = P(t, R(t)) e^{\int_{t+R(t)}^{t+a} \vartheta(s) ds},$$

where $\vartheta(s)$ stands for the adjustment rate in period s .

A formal expression of NDC Systems 2

$$O(t) = \hat{\tau} N \int_{\tilde{R}(t)}^{\tilde{T}(t)} \frac{\int_0^{R(t-a)} \left[W(t-a+b) e^{\int_{t-a+b}^{t-a+R(t-a)} \rho(s) ds} \right] db}{\Gamma(t-a, R(t-a))} e^{\int_{t-a+R(t-a)}^t \vartheta(s) ds} da$$

A formal expression of NDC Systems 2

$$O(t) = \hat{\tau} N \int_{\bar{R}(t)}^{\bar{T}(t)} \frac{\int_0^{R(t-a)} \left[W(t-a+b) e^{\int_{t-a+b}^{t-a+R(t-a)} \rho(s) ds} \right] db}{\Gamma(t-a, R(t-a))} e^{\int_{t-a+R(t-a)}^t \vartheta(s) ds} da$$

- **Crucial task** for the policymaker: Determine the **control variables** $\rho(t)$, $\vartheta(t)$ and $\Gamma(t, R(t))$ in such a way that expenditures develop in line with revenues $I(t) = \hat{\tau} L(t) W(t)$.
- Question: Is this possible for any path of the retirement age $R(t)$ (which is the **choice variable** of the households)?

The first important parameter in NDC systems — The notional interest rate

- Growth rate of **average wages**:

$$\rho(t) = g^W(t) = \frac{\dot{W}(t)}{W(t)}$$

- Growth rate of the **wage bill**:

$$\rho(t) = g^W(t) + g^L(t) = \frac{\dot{W}(t)}{W(t)} + \frac{\dot{L}(t)}{L(t)}$$

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Conventional wisdom: Use the growth rate of the wage bill.

An important caveat

- If the retirement age increases, then the labor force grows – even if the cohort size is constant.

$$L(t) = \tilde{R}(t)N \rightarrow g^L(t) = \frac{\dot{\tilde{R}}(t)}{\tilde{R}(t)}$$

- Increases in the retirement age are, however, necessary to stabilize the dependency ratio $z(t)$. In particular:

$$z(t) = \frac{\tilde{T}(t) - \tilde{R}(t)}{\tilde{R}(t)} = \hat{z} \text{ implies that:}$$

$$\tilde{R}(t) = \frac{\tilde{T}(t)}{1 + \hat{z}} = \frac{T(t)}{(1 + \gamma)(1 + \hat{z})}$$

- In this case: $g^L(t) = \frac{\gamma}{T(t)}$

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→ A third concept for the notional interest rate

- “Life-expectancy adjusted” growth rate of the wage bill:

$$\rho(t) = \frac{\dot{W}(t)}{W(t)} + \frac{\dot{L}(t)}{L(t)} - \frac{\gamma}{T(t)}$$

Example: $\gamma = 0.2$, $T(t) = 60 \rightarrow \frac{\gamma}{T(t)} = 0.33\%$

The second important parameter in NDC systems — Remaining life expectancy

- **Period** (cross-section) life expectancy:

$$\Gamma(t, R(t)) = \tilde{T}(t + R(t)) - R(t)$$

- **Cohort** (forecasted) life expectancy:

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Conventional wisdom: Use cohort life expectancy

Benchmark result — a self-stabilizing budget

Assumptions:

- Constant cohort size
- Linearly increasing life expectancy
- Retirement age proportional to life expectancy:
 $R(t) = \mu T(t)$.

Result:

- A NDC system leads to a **balanced budget** if the following two conditions are fulfilled:
 - The notional interest rate is equal to the **adjusted growth rate of the wage bill**
 - The annuity is calculated by using **period life expectancy**.

$R(t)$ proportional to $T(t)$

For $R(t) = \mu T(t)$ the deficit-ratio $d(t) = \frac{O(t)}{I(t)}$ is given by:

	(1)	(2)
	Notional Interest Rate — Growth Rate of:	
	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	$\approx 1 + \frac{\gamma}{2}$	1
Cohort Life Expectancy	$\approx 1 - \frac{\gamma}{2}$	$\frac{1}{1+\gamma}$

Extensions for various other assumptions about retirement behavior

- $R(t)$ is **constant** → Almost balanced.
- $R(t)$ is **optimally chosen** → (for specific assumptions) balanced or almost balanced.
- $R(t)$ is **random** → Balanced over time.

$R(t)$ is constant

For $R(t) = \bar{R}$ the deficit-ratio $d(t) = \frac{O(t)}{I(t)}$ is given by:

	(1)	(2)
	Notional Interest Rate — Growth Rate of:	
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Period Life Expectancy	$\approx 1 + \frac{\gamma}{2}$	≈ 1
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$R(t)$ is optimally chosen — Case 1

$$\mathbb{U} = \int_0^{T(t)} e^{-\delta a} U(C(t, a)) da - \int_0^{R(t)} e^{-\delta a} V(T(t), a) da,$$

where $V(T(t), a)$ captures the disutility of work of generation t at age a .

Assume: $r = g = \delta = 0$.

Case 1:

- $V(T(t), a)$ is homogeneous of degree 0.
- $R^*(t) = \mu T(t) \rightarrow$ Budget is **always balanced**.

$R(t)$ is optimally chosen — Case 2

Case 2:

- $V(T(t), a) = va$.
- $R^*(t) = \sqrt{\frac{T(t)}{v}} \rightarrow$ Budget is **almost balanced**.

	(1)	(2)
	Notional Interest Rate — Growth Rate of:	
	Wage Bill	Adjusted Wage Bill
Period Life Expectancy	$\approx 1 + \frac{\gamma}{2}$	≈ 1
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$R(t)$ is random

- Retirement age is a random variable. In particular:

$$R(t) = \text{Uniform}(0.53 \times T(t), 0.89 \times T(t)).$$

▶ Example

Conclusions

- A well-designed PAYG system like the NDC system bears the promise to deal successfully with demographic developments.
- In contrast to the conventional wisdom, the most appropriate approach is to use period life expectancy and an adjusted growth rate of the wage bill.
- Given that the retirement behavior cannot be controlled, the system needs a reserve fund to deal with short-run imbalances.
- There are a number of additional factors that might also be potential sources of instability for the system: fluctuations in cohort size, fertility age, in the average age of labor market entry, in the age-earnings profiles or in age-specific mortality.
- It is therefore recommendable that a NDC system includes some additional mechanism that adjusts for unforeseen imbalances like the Swedish “automatic balance mechanism”

Introduction
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Notation and Budget
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NDC systems
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Results
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Conclusions
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Appendix

Appendix

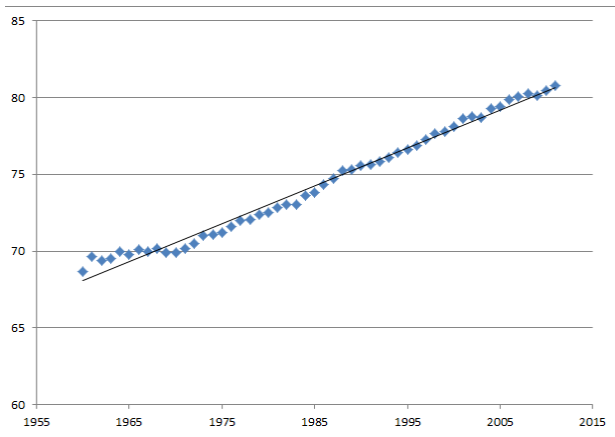
Life Expectancy in the EU

For the EU-countries, e.g., life expectancy at birth is projected to increase over the next 50 years by about 7.5 years which is the main reason behind the projected increase in the old-age dependency ratio from 25.4% in 2008 to 53.5% in 2050 (EPC 2009).

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Life Expectancy in Austria

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(Female) life expectancy from 1840 to present

Oeppen, J. and Vaupel, J. W. (2002). "Broken Limits to Life Expectancy", *Science* 296 (5570).

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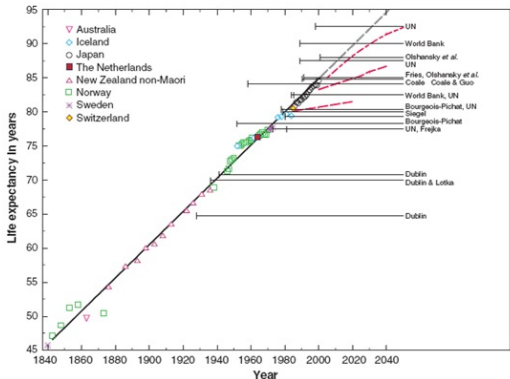


Fig. 1. Record female life expectancy from 1840 to the present [suppl. table 2 (f)]. The linear-regression trend is depicted by a bold black line (slope = 0.243) and the extrapolated trend by a dashed gray line. The horizontal black lines show asserted ceilings on life expectancy, with a short vertical line indicating the year of publication (suppl. table 1). The dashed red lines denote projections of female life expectancy in Japan published by the United Nations in 1986, 1999, and 2001 (f): It is encouraging that the U.N. altered its projection so radically between 1999 and 2001.

Quotes

- Life expectancy increases in a linear fashion:
 - “Because best-practice life expectancy has increased by 2.5 years per decade for a century and a half, one reasonable scenario would be that this trend will continue in coming decades. If so, record life expectancy will reach 100 in about six decades” (Oeppen and Vaupel, 2002).
- Alternative: Life expectancy reaches a maximum age T^{max}

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- Alternative: Life expectancy **reaches a maximum age T^{max}**
 - James Fries (1980): Maximum potential life expectancy is normally distributed around 85 with a SD of 7 years.
 - Olshansky and Carnes (2003): “Organisms operate under warranty periods that limit the duration of life of individuals and the life expectancy of populations”.

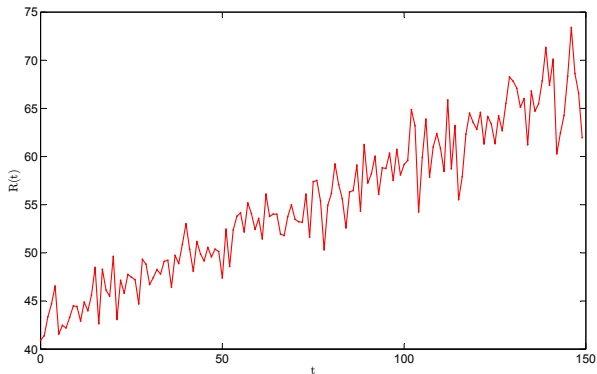
The orange envelope

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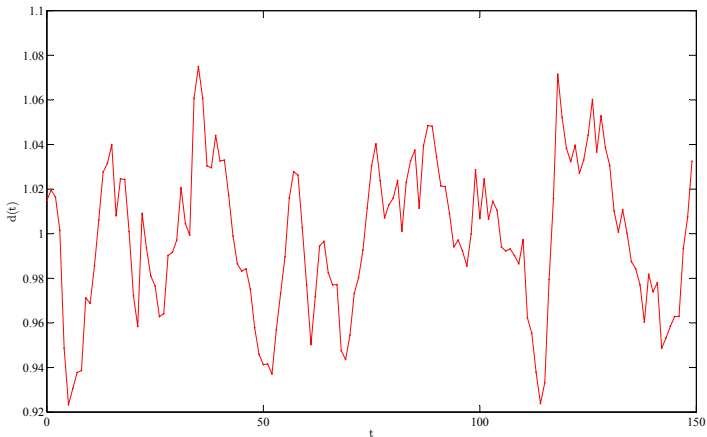
$R(t)$ is a uniform RV between $0.53 \times T(t)$ and $0.89 \times T(t)$

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The deficit ration $d(t) = \frac{O(t)}{I(t)}$ when $R(t)$ is random

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A numerical example to calibrate a pension system

- In a “demographic steady-state”:
 $N(t) = N, T(t) = T, R(t) = R, \forall t.$
- The parameters of the system have to be chosen in a way such that: $\hat{\tau} = \hat{q}\hat{z}$.
- Based on the case of an “average Austrian pensioner”:
 - Retirement age (males): 59.1
 - Life expectancy at the age of 60 (males): 20.7
 - Insured months for new pensioners: 457 (about 38 years)
 - As an approximation this means: the average Austrian pensioner starts working at the age of 20, retires at 60 and dies at 80. Or: $R = 40, T = 60$ and thus $\hat{z} = \frac{60-40}{40} = \frac{1}{2}$.
 - Furthermore: $\hat{\tau} = 0.3$. Why? The contribution rate is (mostly): 22.8%. The “Bundesmittel” in 2010 have been 8.175 Mio. EUR. This is about a third of the income from contributions ($\frac{8.175}{23.496} = 0.35$) and so $22.8 * 1.35 = 30.7\%$.
 - Therefore a “balanced budget” implies: $\hat{q} = \frac{\hat{\tau}}{\hat{z}} = 0.6$.

Is there a maximum potential life expectancy?

- “Although it is likely that anticipated advances in biomedical technology and lifestyle modification will permit life expectancy to continue its slow rise over the short-term, a repetition of the large and rapid gains in life expectancy observed during the 20th century is extremely unlikely” (Carnes and Olshansky, 2003).
- “First, experts have repeatedly asserted that life expectancy is approaching a ceiling: these experts have repeatedly been proven wrong. Second, the apparent leveling off of life expectancy in various countries is an artifact of laggards catching up and leaders falling behind. Third, if life expectancy were close to a maximum, then the increase in the record expectation of life should be slowing. It is not. For 160 years, best-performance life expectancy has steadily increased by a quarter of a year per year, an extraordinary constancy of human achievement” (Oeppen and Vaupel, 2002).

Results when life expectancy has an upper limit

Results when life expectancy has an upper limit

Is there a maximum potential life expectancy? Is it constant over time? And how far away from this limit are we right now?

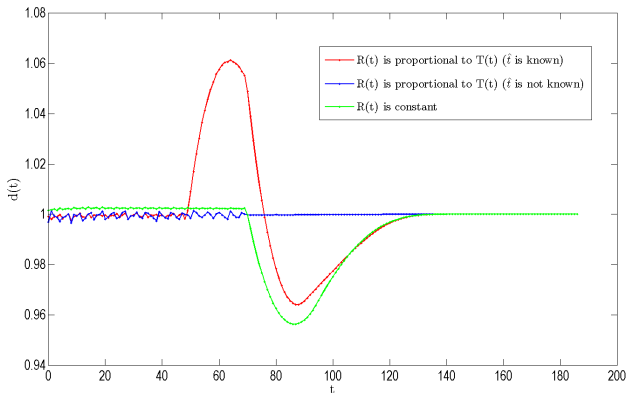
▶ More quotes

$$T(t) = T(0) + \gamma \cdot t, \text{ for } t < \hat{t}$$

$$T(t) = T(\hat{t}) = T^{max}, \text{ for } t \geq \hat{t}$$

▶ Graph

$d(t)$ when cohort life expectancy reaches a limit



Related literature

- Main result in **contrast** to Torben Andersen (JPubE, 2008): “An indexation of pension ages to longevity may seem a simple and fair solution. This would imply that the relative amount of time spent as contributor to and beneficiary of a social security scheme would be the same across generations with different longevity. [...] However, as is shown in this paper, this solution is not in the feasibility set”
- **Reason:** TA works with a “two life phases” model where the first life phase has length 1, the second phase has length $\beta \leq 1$ and individuals retire at age $\alpha \leq \beta$.
- The relative retirement age is defined as $\frac{\alpha_t}{\beta_t}$ which is somewhat unusual. Using $\frac{1+\alpha_t}{1+\beta_t}$ leads to the same result as in my framework.